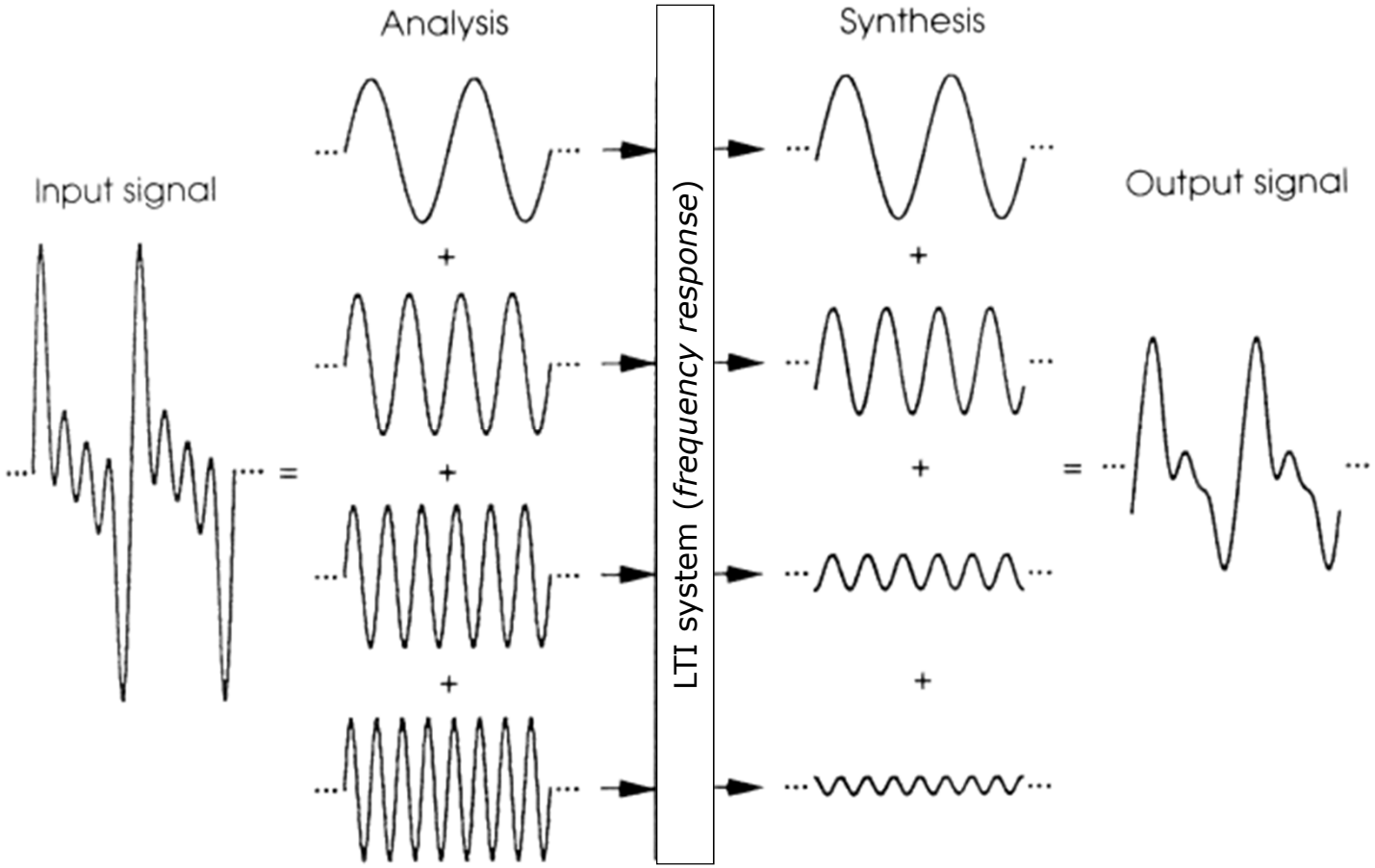
Signals, systems, acoustics and the ear

Week 5

Impulse responses

Again!



waveform → *spectrum*

Fourier analysis

spectrum → *waveform*

Fourier synthesis

What you know about so far

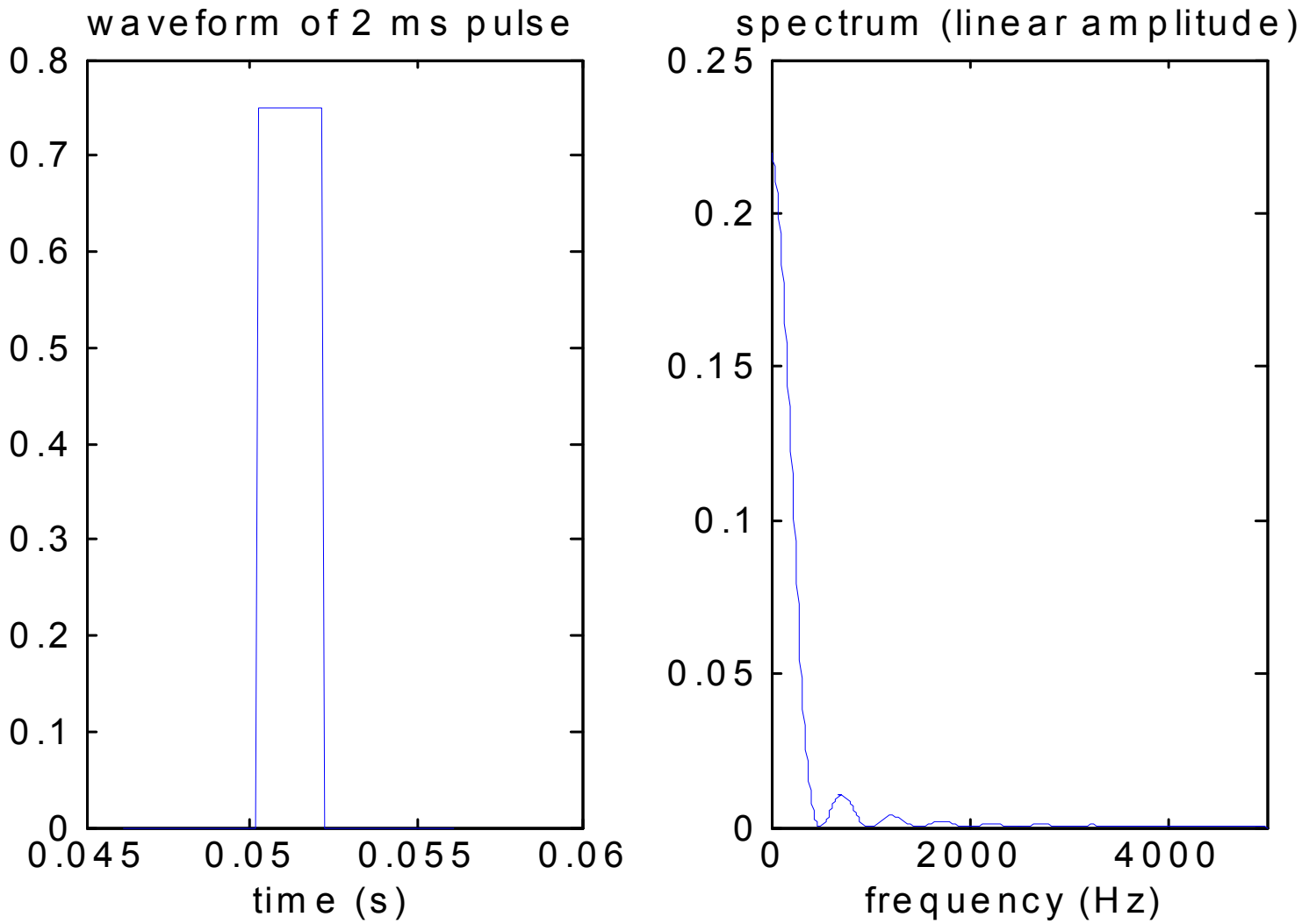
	time domain *(= time on the x axis)*	*frequency domain* *(= frequency on the x axis)*
signals		
systems		

What is an impulse response?

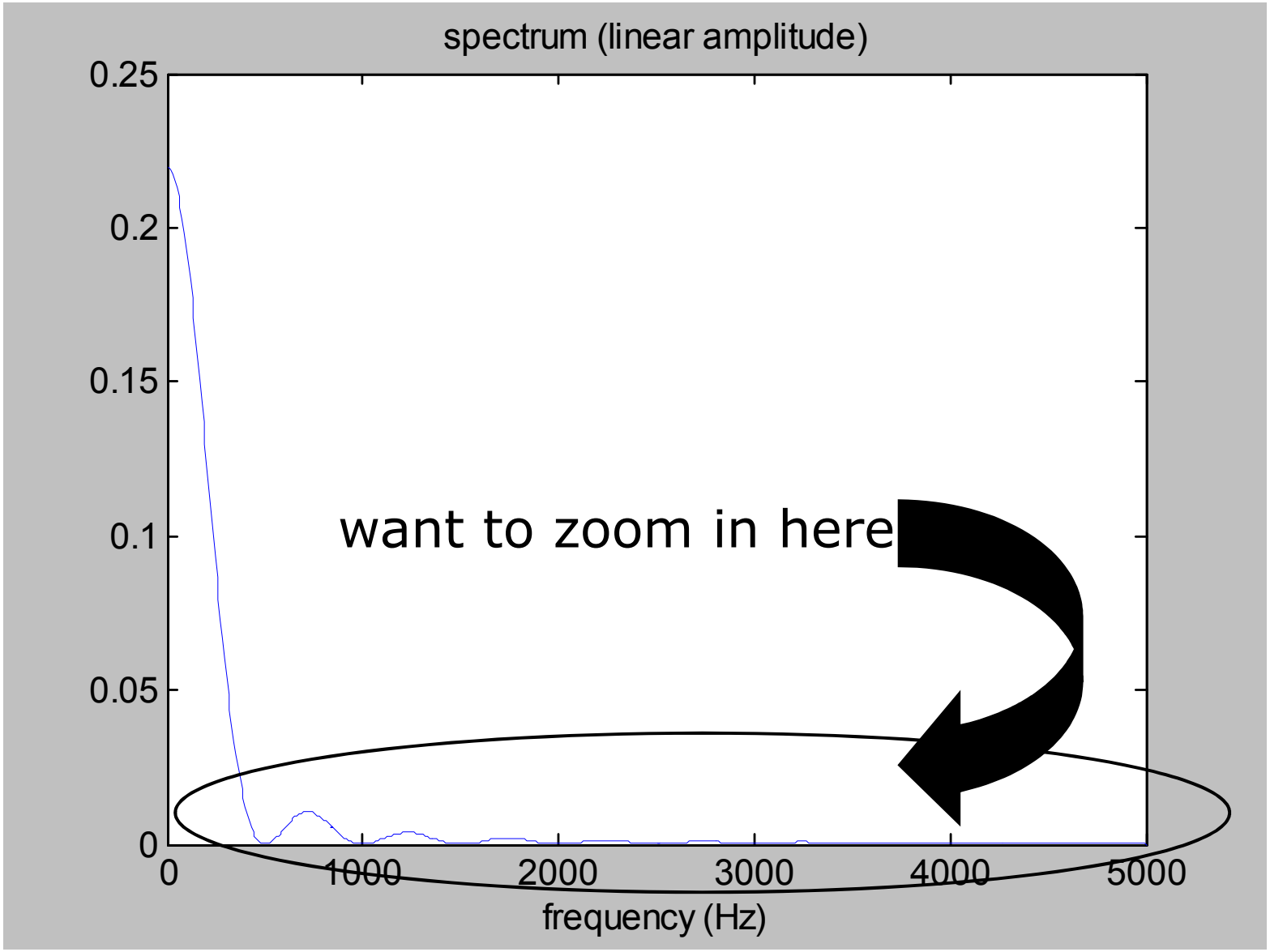
- The response of a system to an impulse
- So what's an impulse?
  - A special kind of wave
  - The Platonic ideal rectangular pulse that is …
    - incredibly narrow, and incredibly tall
  - A genuine impulse can't exist …
    - but you can get as close as you need

Modifying a rectangular pulse (which can exist) to make an impulse (which cannot)

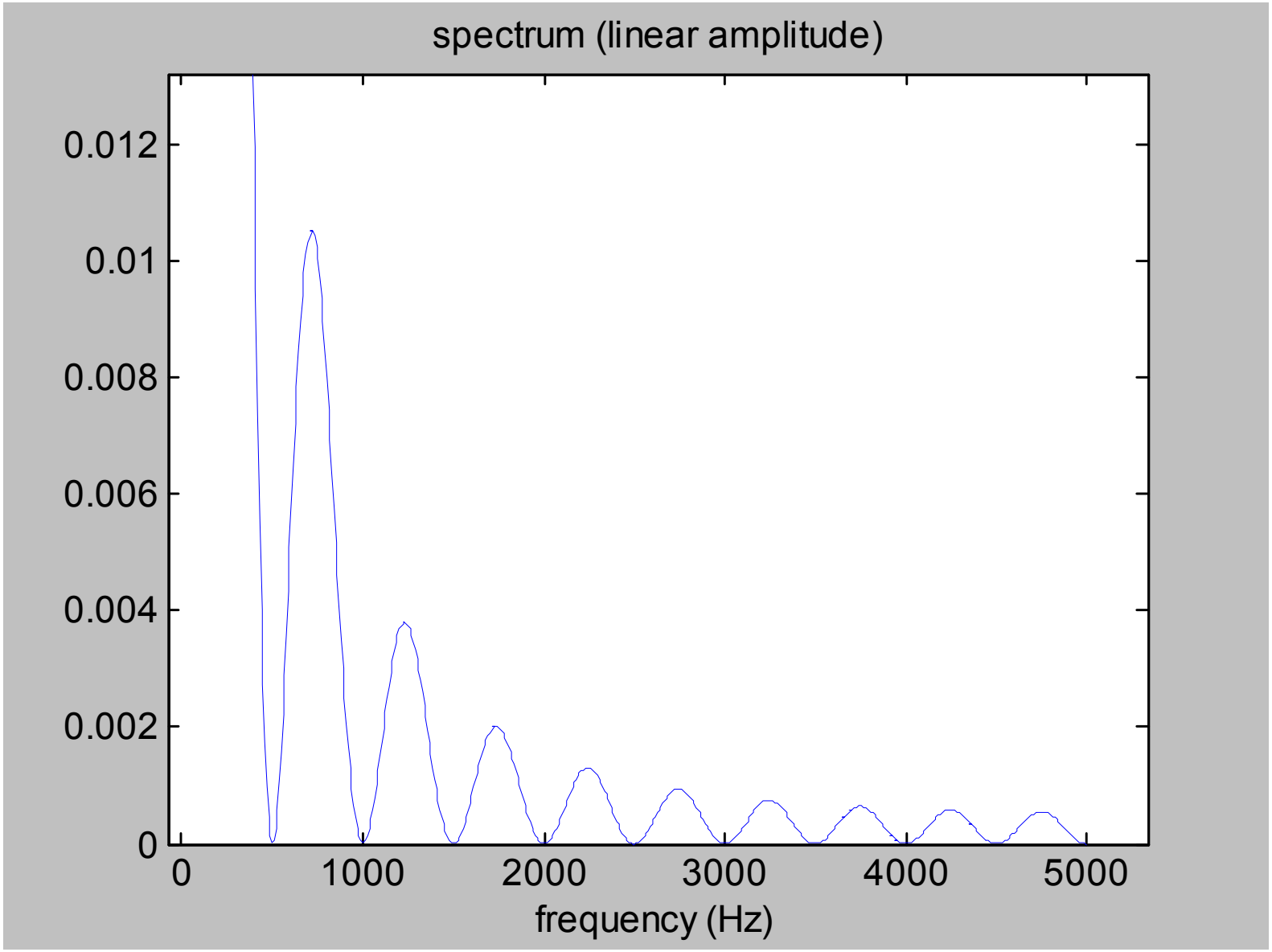
Waveform and spectrum of a 2 ms pulse



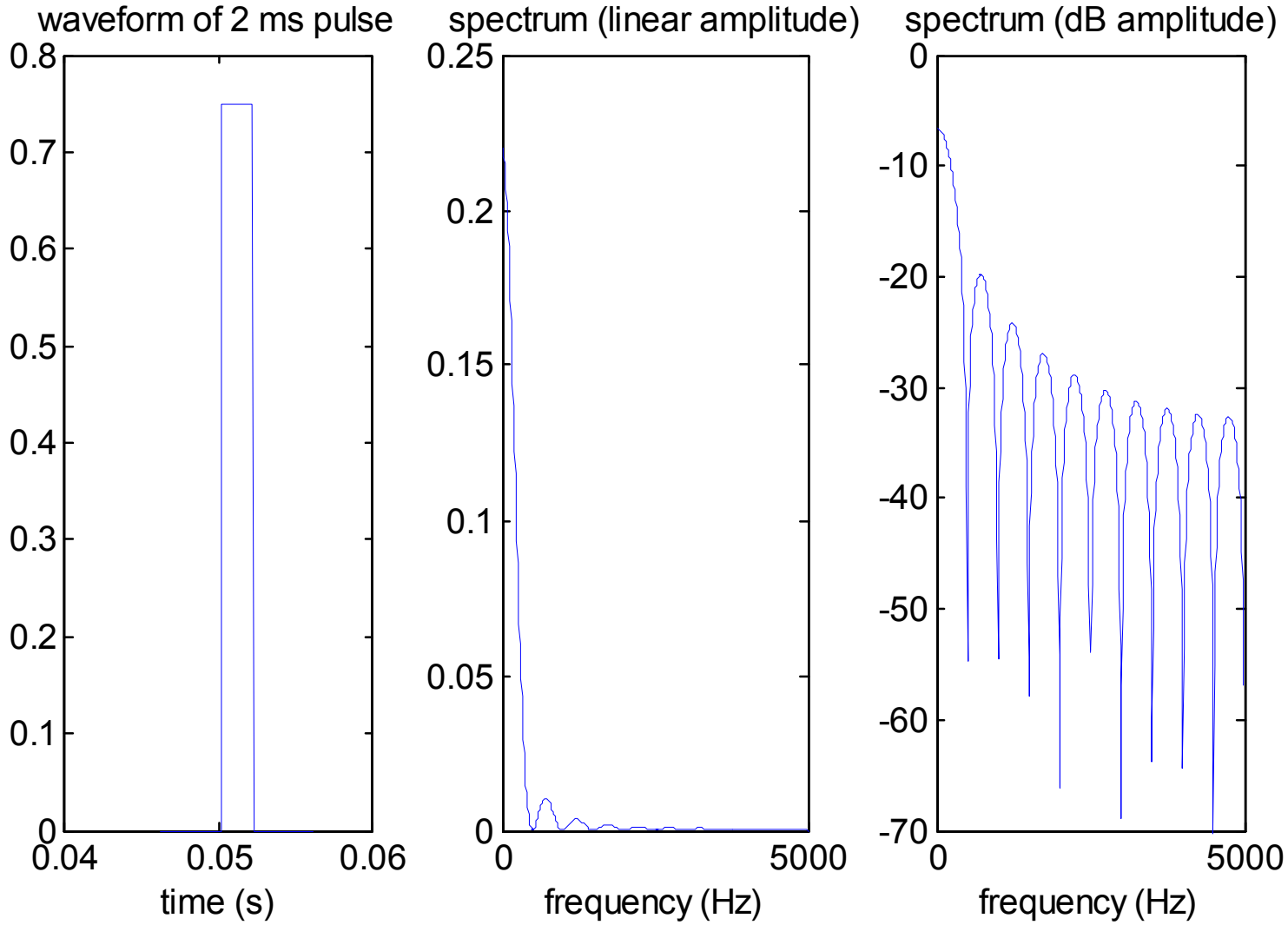
Looking carefully at the spectrum



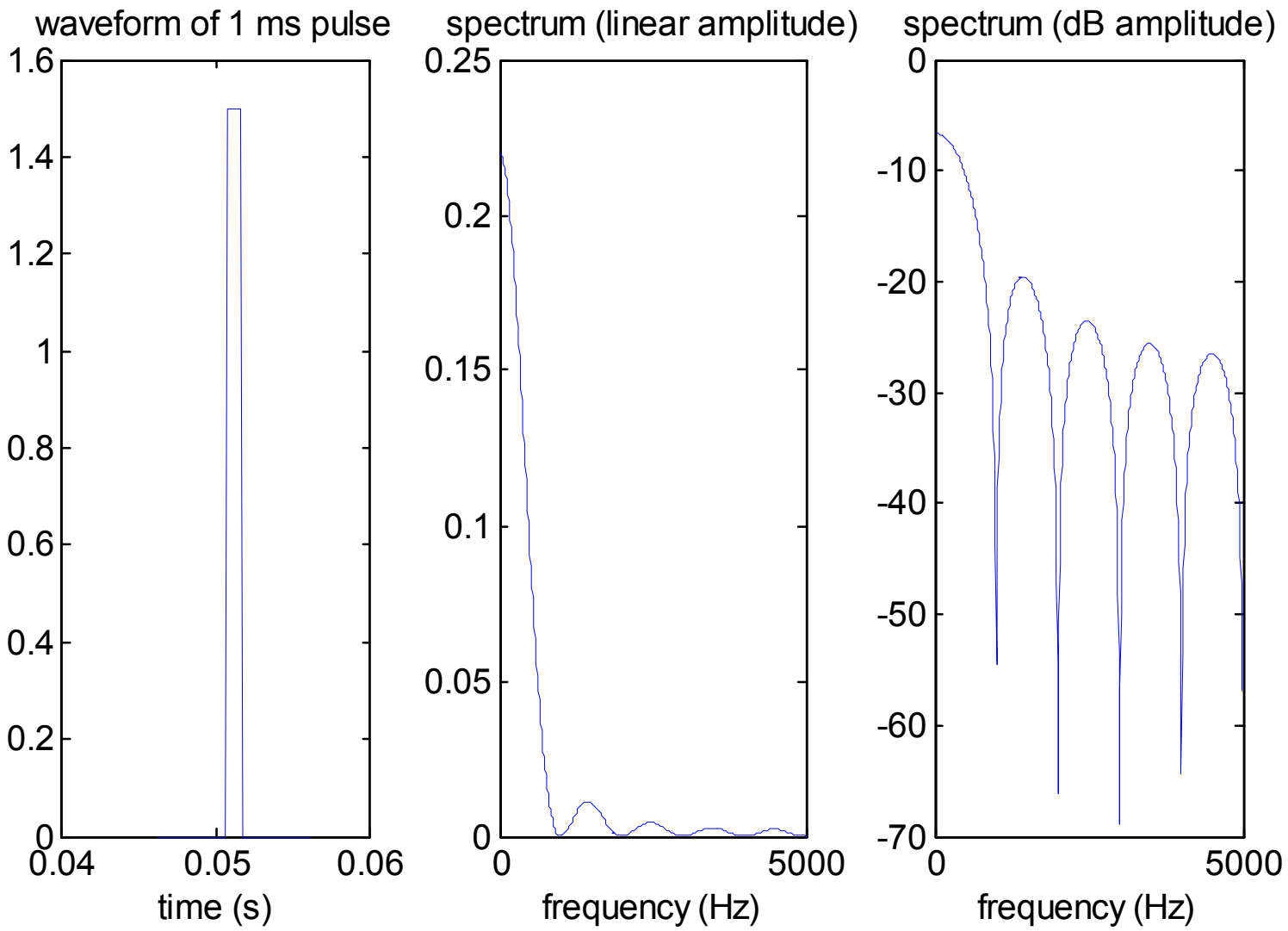
Looking carefully at the spectrum



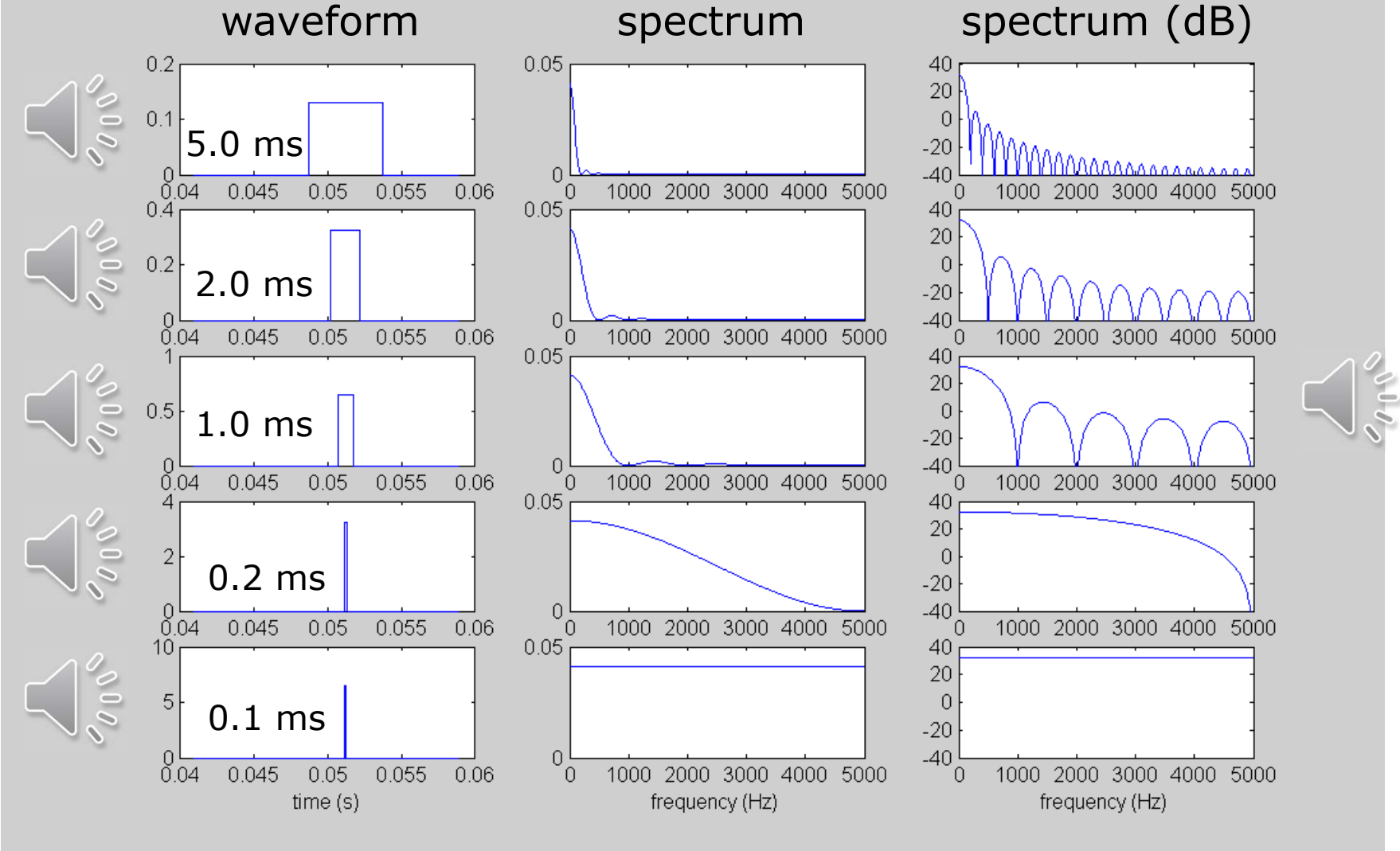
Easier to see spectrum on a dB scale: most of the energy is within the first ‘lobe’



Amplitude spectrum of a 1 ms pulse



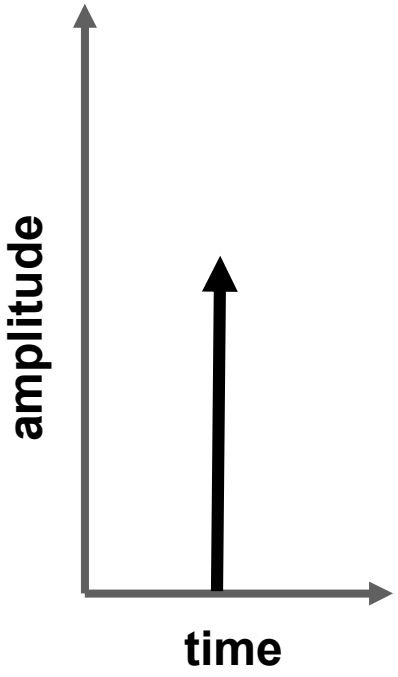
Spectrum of a rectangular pulse (zeros at multiples of 1/pulse duration)



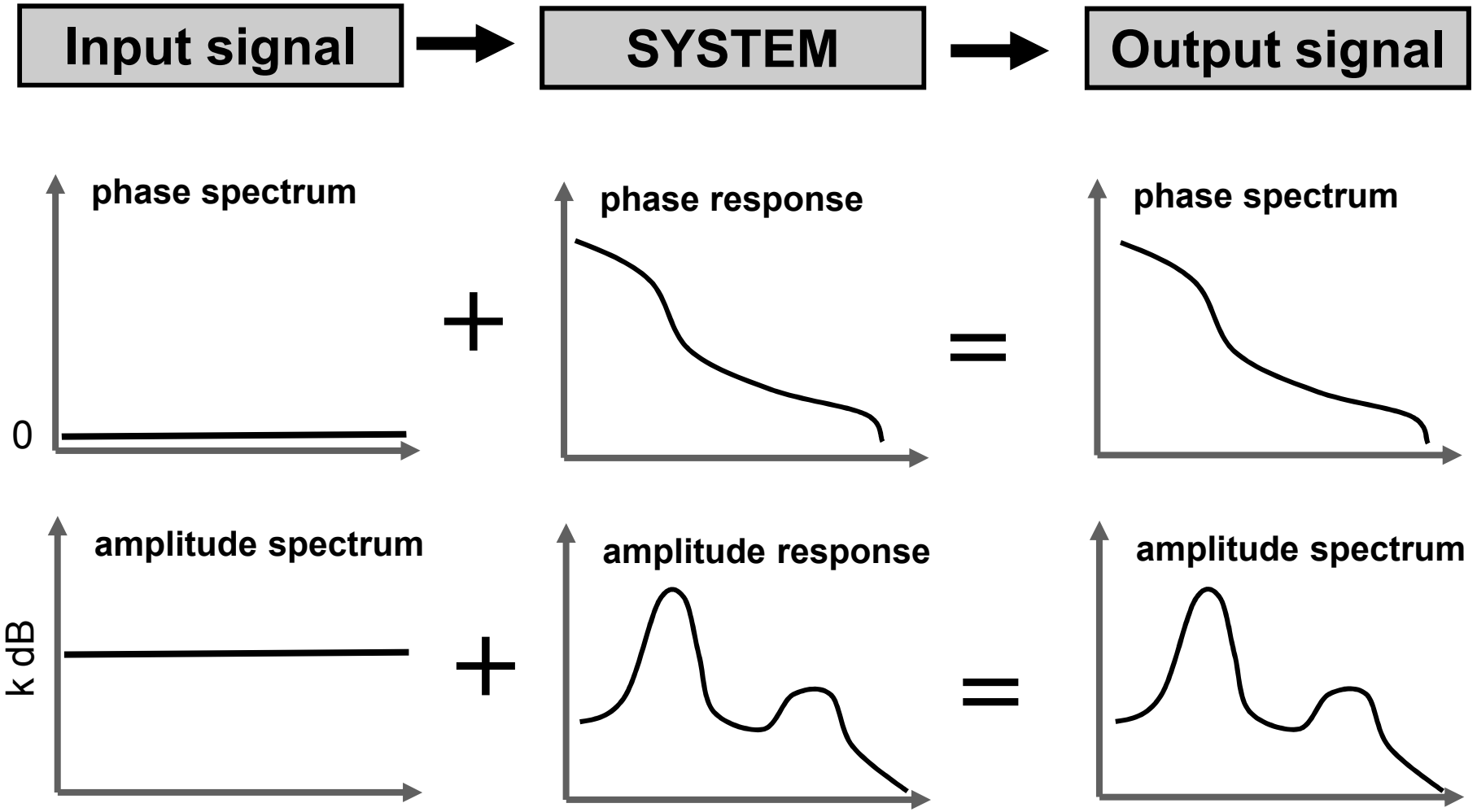
The spectrum of an impulse (infinitely narrow rectangular pulse)

- amplitude spectrum
  - flat (equal amplitude at all frequencies)
  - what other signal has this property?
  - so what else must be different?
- phase spectrum
  - 0 everywhere (when impulse is at time=0)

Response of an LTI system to an impulse



Response of an LTI system to an impulse



The spectrum of the impulse response has the same shape as the frequency response of the system

In simple equations, for any LTI system ('f'=frequency) ...

- Amplitude (on dB scales)

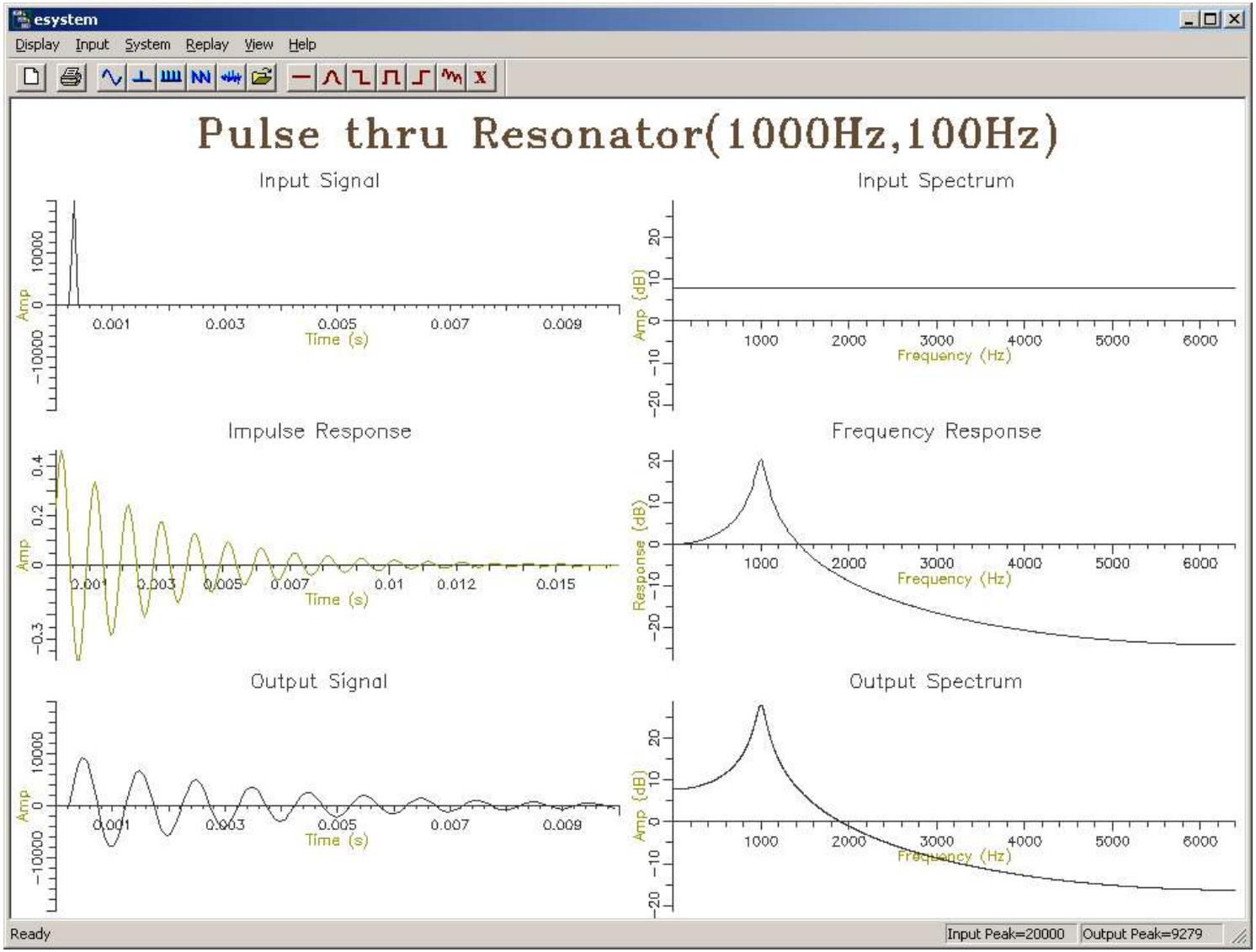
  Output amplitude (f) = Input amplitude (f) + gain(f)
- Phase

  Output phase (f) = Input phase (f) + Phase response(f)
- But …
  - Input amplitude (f) = a constant (k dB)
  - Input phase (f) = 0
- So …

  Output amplitude (f) = gain(f) + k

  Output phase (f) = Phase response(f)

An impulse response



Interesting fact

- Converting a *waveform* $\rightarrow$ *spectrum* is exactly the same process as converting *impulse response* $\rightarrow$ *frequency response*
  - A Fourier analysis

Summary

- An infinitesimally narrow and infinitely high pulse of finite energy is known as an ***impulse***
  - a mathematical abstraction
- An impulse response tells you everything you need to know about an LTI, because an impulse contains all frequencies equally
  - The amplitude spectrum of the impulse response is simply the amplitude response of the system (and the same for phase)
  - A ***time-domain*** characterisation of an LTI
  - ***Convolution*** is the process which allows the calculation of the output signal from a given input signal and the impulse response